# **Lesson Objectives**

1. Writing and Graphing Inequalities
2. Interval Notation – Overview
3. Convert among inequality, graph, and interval notation (any variation)
   * **Non-ending** interval (involves positive or negative infinity)
4. Compound Inequalities
   * **Open or Closed** interval (in BETWEEN 2 endpoints – infinity is NOT involved)
   * **Two non-ending intervals** together (2 endpoints going AWAY from each other)

# **Writing & Graphing Inequalities**

## **Warm-Up**

**Directions:** Compare. Write <, >, or = in the blank.

1. – 3 \_\_\_\_ 2
2. 6.5 \_\_\_\_ 6.3
3. \_\_\_\_
4. 0.25 \_\_\_\_

## **Graphing Inequalities**

To write and graph inequalities, you must connect words with symbols.

Look at the information in the table below.

|  |  |  |
| --- | --- | --- |
| **Symbol** | **Words** | **Graph** |
|  | “is \_\_\_\_\_\_\_\_\_\_\_\_\_\_ than” | Shades to the \_\_\_\_\_\_\_\_\_  Endpoint \_\_\_\_CLUDED |
|  | “is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ than or equal to”  “at most”  “no more than” | Shades to the \_\_\_\_\_\_\_\_\_  Endpoint \_\_\_\_CLUDED |
|  | “is \_\_\_\_\_\_\_\_\_\_\_\_\_\_ than” | Shades to the \_\_\_\_\_\_\_\_\_  Endpoint \_\_\_\_CLUDED |
|  | “is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ than or equal to”  “at least”  “no less than” | Shades to the \_\_\_\_\_\_\_\_\_  Endpoint \_\_\_\_CLUDED |

# **Interval Notation - Overview**

* Domain and range do not always involve a **discrete** (\_\_\_\_\_\_\_\_\_\_\_) number of elements.
* **Interval notation** is used for **\_\_\_\_\_\_\_\_\_\_\_\_\_\_** elements (not countable).
* Set-builder notation (\_\_\_\_\_\_\_\_\_\_\_\_\_\_) can convert to interval notation, and vice-versa.

**Interval Notation** is describing how the number line is \_\_\_\_\_\_\_\_ (or “painted”), written as:

* the \_\_\_\_\_\_\_\_\_\_\_\_ value first, and
* the \_\_\_\_\_\_\_\_\_\_\_\_ value second.

It’s identifying the starting and ending points of the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.   
\_\_\_\_\_\_ points in \_\_\_\_\_\_\_\_\_\_\_\_\_ are shaded (included).

When using interval notation, the starting and ending points also contain a symbol:

**(** or **)** means “not included” or “open”

**[** or **]** means “included” or “closed”

## **General format for Interval Notation**

The general format for Interval Notation looks like this:

**\_\_ or \_\_ \_\_\_\_\_\_\_\_ number, \_\_\_\_\_\_\_\_ number \_\_ or \_\_**

Note that the smaller number could be , or the larger number could be . Remember to ALWAYS use \_\_\_\_\_\_\_\_\_\_\_\_\_ with either positive or negative \_\_\_\_\_\_\_\_\_\_\_\_.

**Non-ending interval:** is interpreted as , where *a* \_\_\_\_\_\_\_\_\_\_\_ included and infinity is always expressed as being “open” (not included).

Example: graph: 

Interval Notation:

**Non-ending interval:** is interpreted as , where *b* \_\_\_\_\_\_\_\_\_\_ included and again, infinity is always expressed as being “open” (not included).

Example: graph: 

Interval Notation:

# **Convert among Inequality, Graph, and Interval Notation (any variation)**

## **Non-ending interval (involves positive or negative infinity)**

* **EXAMPLE:** Write the set in interval notation. [2.3.1]

[SOLUTION] The expression there at the end is read as:

“The set of all values *x*, \_\_\_\_\_\_\_\_\_\_\_\_\_\_ *x* is \_\_\_\_\_\_\_\_\_\_\_\_\_\_ than 5.”

That expression is written in **\_\_\_\_\_\_\_\_-\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ notation**.

For ease of use, you can \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ the braces and the initial “” part.

Focus just on the INEQUALITY part:

* Ensure the variable is on the **\_\_\_\_\_\_\_\_\_\_** of the symbol (it IS)
  + If it isn’t, you need to **REVERSE** the inequality (not needed here):
  + By far the most common error students make – forgetting to reverse it.
* is read as “*x* is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ than 5”
* First, look at the **graph** of this inequality:



* What is the SMALLER number?
  + Is it INCLUDED or EXCLUDED? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
  + How do you know? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
  + Use BRACKET or PARENTHESIS? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* What is the LARGER number?
  + Is it INCLUDED or EXCLUDED? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
  + How do you know? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
  + Use BRACKET or PARENTHESIS? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
    - Interval notation is always done as: **[** or **( smaller number , larger number )** or **]**

ANSWER: Given the set-builder notation,

In interval notation:

* **EXAMPLE:** Write the interval notation for the graph. [\*Beecher JIT.6.8]



[SOLUTION]

* What is the SMALLER number?
  + Is it INCLUDED or EXCLUDED? \_\_\_\_\_\_\_\_\_\_\_\_\_
  + How do you know? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* What is the LARGER number?
  + Is it INCLUDED or EXCLUDED? \_\_\_\_\_\_\_\_\_\_\_\_\_
  + How do you know? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
  + Use BRACKET or PARENTHESIS? \_\_\_\_\_\_\_\_\_\_\_\_\_\_
    - Interval notation is always done as: **[** or **( smaller number , larger number )** or **]**

In interval notation:

* **EXAMPLE:** Write the interval notation for the graph. [\*Beecher JIT.6.10]



[SOLUTION]

* What is the SMALLER number?
  + Is it INCLUDED or EXCLUDED? \_\_\_\_\_\_\_\_\_\_\_\_\_\_
  + How do you know? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* What is the LARGER number?
  + Is it INCLUDED or EXCLUDED? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
  + How do you know? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
    - Interval notation is always done as: **[** or **( smaller number , larger number )** or **]**

In interval notation:

(go on to the next page)

* **EXAMPLE:** Write the inequality in interval notation. [1.3.5]

The expression there at the end is read as:

“The set of all values *x*, such that negative 3 is \_\_\_\_\_\_\_\_\_\_\_\_\_ than or equal to *x*.”

That expression is written in **\_\_\_\_\_\_\_-builder notation**.

For ease of use, you can IGNORE the braces and the initial “” part.

Focus just on the INEQUALITY part:

* Ensure the variable is on the **LEFT** of the symbol (it’s \_\_\_\_\_\_\_\_).
  + If it isn’t, you need to **\_\_\_\_\_\_\_\_\_\_\_\_\_** the inequality:
  + Keep the “\_\_\_\_\_\_\_\_\_\_” of the inequality pointed to the \_\_\_\_\_\_\_\_\_ object.
  + By far the **most common error students make – \_\_\_\_\_\_\_\_\_\_\_\_\_ to reverse it**.
* is read as “*x* is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ than or equal to negative 3”
* First, look at the **graph** of this inequality:



* What is the SMALLER number? \_\_\_\_\_
  + Is it INCLUDED or EXCLUDED? \_\_\_\_\_\_\_\_\_\_\_\_\_\_
  + How do you know? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
  + Use BRACKET or PARENTHESIS? \_\_\_\_\_\_\_\_\_\_\_\_\_\_
* What is the LARGER number?
  + Is it INCLUDED or EXCLUDED? \_\_\_\_\_\_\_\_\_\_\_\_\_\_
  + How do you know? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
  + Use BRACKET or PARENTHESIS? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
    - Interval notation is always done as: **[** or **( smaller number , larger number )** or **]**

ANSWER: Given the set-builder notation,

In interval notation:

# **Compound Inequalities**

## **Open or Closed interval (in \_\_\_\_\_\_\_\_\_\_\_\_\_\_ two endpoints – infinity is NOT involved)**

* **EXAMPLE:** Write the interval notation for the set [\*Beecher JIT.6.6]

[SOLUTION]

That expression is written in **set-builder notation**.

For ease of use, you can IGNORE the braces and the initial “” part.

Focus just on the INEQUALITY part:

This is one type of **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ INEQUALITY**, because it involves more than one endpoint. Notice that the variable is **IN \_\_\_\_\_\_\_\_\_\_\_\_\_\_** the two endpoints.

There is a common structure with this “in-between” inequality:

* The \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ number is always on the \_\_\_\_\_\_\_\_\_\_\_\_
* The \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ number is always on the \_\_\_\_\_\_\_\_\_\_\_\_
* (This mimics how they truly are on the number line as well.)
* \_\_\_\_\_\_\_\_\_\_ symbols are pointing \_\_\_\_\_\_\_\_\_ (\_\_\_\_\_\_\_\_\_\_\_\_\_\_-than type)
  + The symbols can use any combination of or .

Returning to the inequality:

Pull these apart into 2 separate inequalities: and

Let’s graph these TOGETHER on the \_\_\_\_\_\_\_\_\_\_\_ number line.

\_\_\_\_\_\_\_\_\_\_\_ ***before*** you graph it: reverses to

You need to graph TOGETHER these 2 inequalities: and

– 10 (\_\_\_cluded) shading \_\_\_\_\_\_\_\_\_\_ and +10 (\_\_\_cluded) shading \_\_\_\_\_\_\_\_\_\_



ANSWER: The set in interval notation is:

You can graph compound inequalities MUCH \_\_\_\_\_\_\_\_\_\_\_\_ than the previous example, by streamlining the process. Let’s look at another example:

* **EXAMPLE:** Write the inequality in interval notation. [1.3.3]

[SOLUTION]

The smaller number – 3 has the symbol < (– 3 \_\_\_cluded = \_\_\_\_\_\_\_\_\_\_\_)

The larger number 2 has the symbol ≤ (2 \_\_\_cluded = \_\_\_\_\_\_\_\_\_\_\_\_)

**ANSWER**: The inequality in interval notation is

* **EXAMPLE:** Write the inequality in interval notation. [1.3.9]



[SOLUTION]

This time the inequality is not given – only the graph is given. This is even easier!

* Smaller number = – 7 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (\_\_\_\_cluded)
* Larger number = 4 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (\_\_\_\_cluded)

**ANSWER**: That inequality as graphed in interval notation is

## **Two non-ending intervals** together (2 endpoints going \_\_\_\_\_\_\_\_\_\_\_ from each other)

The other type of COMPOUND INEQUALITY is where the shading starts at 2 endpoints and they go \_\_\_\_\_\_\_\_\_ from each other. A big giveaway is that you’ll see the word **“\_\_\_\_\_”** or the “\_\_\_\_\_\_\_\_\_\_\_\_” symbol, .

* **EXAMPLE:** Express the set in interval notation. [1.3.7]

[SOLUTION]

There are 2 separate inequalities separated by **“or”**:

Make sure the \_\_\_\_\_\_\_\_\_ endpoint is written on the \_\_\_\_\_, the \_\_\_\_\_\_\_\_ on the \_\_\_\_\_\_.

Graph these 2 inequalities TOGETHER on the \_\_\_\_\_\_\_\_\_ number line:



**Left piece Right piece**

Smaller number: (\_\_\_\_\_\_\_\_\_\_\_) Smaller number: \_\_\_ (\_\_\_\_\_\_\_\_\_\_\_)

Larger number: \_\_\_\_ (\_\_\_\_\_\_\_\_\_\_\_) Larger number: (\_\_\_\_\_\_\_\_\_\_\_)

Interval notation: Interval notation:

MERGE these two pieces TOGETHER, remove the word “or” and use “\_\_\_\_\_\_\_\_” symbol **()**:

ANSWER: The interval notation for is

Sources used:

1. Desmos online graphing calculator, located at [www.desmos.com/calculator](http://www.desmos.com/calculator)
2. Pearson *MyLab Math*: *College Algebra with Integrated Review*, *5th Edition*, Beecher
3. Pearson *MyLab Math*: *College Algebra with Modeling and Visualization, 6th Edition*, Rockswold